

Geometric Modeling in Computer – aided Geometric Design

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Dedication

I want to dedicate this book to Adriana and Simon, for their love and patience. C. A. C. M.

Thanks to my parents, brothers, sisters, professors, teachers, who taught me with patience and affection the love for independent and formal thinking, and thanks to Maria Stella, Pastora and friends, who try very hard to civilize me. O. E . R. S.

Chapter 1

Introduction

The domain of Computer Aided Design continuously changes, incorporating new technologies. However, a rough subdivision of CAD can be made into: (1) Geometric Transformations, (2) Parametric Curves and Surfaces, and (3) Geometric Modeling. This book visits Geometric Modeling. This domain seeks to give formal geometric representation to the usual objects in engineering, arts, medicine, among other fields. Geometric Models internally use both Geometric Transformations and Parametric Curves and Surfaces, among many other mathematical methods.

However, Geometric Modeling is the domain in which a CAD Software end-user can make modeling decisions that will be reflected in most immediate manner in the quality of the data, agility of import / export processes, and subsequent usage in FEA, Manufacturing, Visualization, Prototyping, etc. The other domains (1 and 2 above) run in the interior of a Geometric Modeler, but the end - user usually has little opportunity to affect functioning or decisions of such methods.

This book intends to give the reader the theoretical foundations to understand and solve the problems that arise when using CAD data in downstream applications (e.g. Computational Mechanics). By under-

standing the theory, for example the concept of Manifoldness, the CAD user will understand the difficulty in exporting, meshing and solving a FEA problem. Such an understanding will allow this user to adopt correct design practices, which will save up to 90 % of the resources spent in the trial - and - error process of Design vs. Usage of the Design.

The various Geometric Modeling techniques (Boundary Representation, Constructive Solid Geometry, Design Intent, Enumerations, Meshing, etc.) obviously interplay with each other and such conversions are precisely which amplify their application domain. This book intends to give formal (although intuitive) mathematical definitions, as needed to understand the important points. The book avoids an overly mathematical treatment of the concepts. In similar manner, this book does not go into the conversion among the modeling schema above, because it has combinatorial size, and because in many cases such conversions are not well defined in the mathematical domain (which does not impede that they exist in the real - life engineering applications).

The examples in the book have been explained to generations of students in U. EAFIT, in the courses Introduction to CAD CAM Systems, Geometric Modeling, Underlying Mathematics for CAD CAM, Computer Aided Geometric Design, and others. The authors decided to compile and order this material, and have directed the solution and edition of such examples, iteratively improving them, in an never - ending process (as corrections frequently bring new imperfections). The authors feel that these examples have reached a point in which the editorial quality allows their presence in this book. The authors wish to thank this stream of collaboration, in the many editions of the above courses. The citations of such collaborations appear in each example, and in the Reference and the Acknowledgment sections.